

5. Let $h \in \text{in}_{<}(I)$, since $\{g_1, \dots, g_m\}$ is a Groebner basis for I then

$$h = q_1 \text{in}_{<}(g_1) + \dots + q_m \text{in}_{<}(g_m)$$

distributing the monomials $\text{in}_{<}(g_i)$ in the polynomials q_i we obtain a sum of monomials such that each monomial is divisible by $\text{in}_{<}(g_i)$ for some i .

Now let $h \in R$ and $h = m_1 + \dots + m_k$ such that each m_i is divisible by $\text{in}_{<}(g_j)$ for some j . This means $h = m'_1 \text{in}_{<}(g_{i_1}) + \dots + m'_k \text{in}_{<}(g_{i_k})$ for $i_r \in \{1, \dots, m\}$ so $h \in \text{in}_{<}(I)$