(c) The weighted ordering of basis $B = \{v_1, \ldots, v_n\}$ of R^n is a monomial ordering if and only if for every $k \in [n]$ the smallest v_j whose k-th is non-zero [7] has a positive k-th coordinate. We proof both directions separately.

$\Rightarrow$) Suppose that the weighted ordering "$<$" of B is a monomial ordering. Let $k \in [n]$ and consider the monomial x_k. Since "$<$" is a monomial order $x_k > 1$. It follows that the first vector v_i such that $\text{multideg}(x_k) \cdot v_i \neq 0$ must contain a positve k-th entrance, because the dot product is supposed to be greater than 0 (otherwise we have that $1 > x_k$) since the only non-zero entrance of $\text{multideg}(x_k)$ is the k-th one. The result follows.

$\Leftarrow$) Suppose that for every $k \in [n]$ the smallest v_j whose k-th is non-zero has a positive k-th coordinate. Let $m \neq 1$ be a monomial. Note that the smallest vector v_i such that $\text{multideg}(m) \cdot v_i \neq 0$ contains only nonnegative entries on the entries where $\text{multideg}(m)$ has non-zero entries (becuase it is the first time a non-zero number appears on those special coordinates). Therefore $m \geq 1$. Suppose now that $m_1 > m_2$ and let m be any monomial. We have that $\text{multideg}(mm_i) = \text{multideg}(m) + \text{multideg}(m_i)$ so $\text{multideg}(mm_1) \cdot v_i - \text{multideg}(mm_2) \cdot v_i = \text{multideg}(m_1) \cdot v_i - \text{multideg}(m_2) \cdot v_i$. So the v_i thus $mm_1 > mm_2$ because the smallest vector that makes different m_1 and m_2 is the smallest vector that makes mm_1 and mm_2 different. It remains to prove that the order relation defined is total. Let A the $n \times n$ matrix whose $k - th$ row is v_k and consider the map $f : \mathbb{R}^n \to \mathbb{R}^n$ given by $f(x) := Ax$. This is a bijection because B is a basis for $\mathbb{R}^n$. But the k-th entrance of the new vector Ax is $v_k \cdot x$. AS a particular result we obtain that for every pair of distinct vectors u_i, u_j in $\mathbb{Z}^n_{\geq 0}$ there is some $v_k \in B$ such that $u_i \cdot v_k \neq u_j \cdot v_k$ (by the bijectivity of f), yielding a way to compare both vectors on the defined order. So the order is total and we are done.