some problems i have proposed.

federico ardila

2002 International Mathematical Olympiad, Problem 1.

Let n be a positive integer. Let T be the set of points (x, y) of the plane, with x, y non-negative integers such that $x + y < n$. Each element of T is colored red or blue, so that if (x, y) is red and $x' \leq x, y' \leq y$, then (x', y') is also red. Let an X-set be a set of n blue points with pairwise different x-coordinates. Let a Y-set be a set of n blue points with pairwise different y-coordinates.

Show that the number of X-sets is equal to the number of Y-sets.

2001 International Mathematical Olympiad shortlist.
2002 Colombian Mathematical Olympiad, Problem 3.

Let $A = (a_1, a_2, \ldots, a_{2001})$ be a sequence of positive integers. Let m be the number of sets $\{i, j, k\}$ with $1 \leq i < j < k \leq 2001$ such that $a_j = a_i + 1$ and $a_k = a_j + 1$.

Considering all such sequences A, find the greatest possible value of m.

2000 International Mathematical Olympiad shortlist.
2001 Colombian Mathematical Olympiad, Problem 3.

Let $n \geq 4$ be a positive integer. Given a set $S = \{P_1, \ldots, P_n\}$ of points in the plane such that no three are collinear and no four are concyclic, let a_t be the number of circles $P_iP_jP_k$ which contain point P_t in their interior, and let $m(S) = a_1 + \cdots + a_n$.

Prove that, knowing only the values of n and $m(S)$, it is possible to determine whether the points of S are the vertices of a convex polygon.

2001 Iberoamerican Mathematical Olympiad for University Students, Problem 5.

Let $A \in \mathbb{R}^{n \times n}$ be a matrix and $x \in \mathbb{R}^n$ be a vector such that $A^2x = x$. If all the entries of A and x are positive real numbers, prove that $Ax = x$.

1999 Asian Pacific Mathematical Olympiad, Problem 5.

Let S be a set of $2n+1$ points, no three collinear and no four concyclic. A circle will be called **good** if it has three of the given points on its circumference, $n-1$ of them inside it, and the remaining $n-1$ outside it.

Prove that the number of good circles has the same parity as n.

Note. An old chinese problem asked to show that there is always at least one good circle; this problem asked to show that the number of them has the same parity as n...

In fact, the number of good circles is always equal to n^2! See my article “The number of point-splitting circles”, which is available on my website: `http://www.mit.edu/~fardila/www/` for more information.

1997 Iberoamerican Mathematical Olympiad, Problem 6.

Given n points $P_1, P_2, \ldots, P_n$ inside a circle of radius 1, one of which is the center of the circle. For $1 \leq i \leq n$, let x_i be the distance from P_i to the point P_j $(j \neq i)$ closest to it.

Prove the inequality $x_1^2 + x_2^2 + \ldots + x_n^2 \leq 9$.

1997 Iberoamerican Mathematical Olympiad, Problem 1.

Let $r > 0$ be a real number such that the following condition holds. If m and n are integers such that n is a multiple of m and $\lfloor mr \rfloor \geq 1$, then $\lfloor nr \rfloor$ is a multiple of $\lfloor mr \rfloor$.

Prove that r is an integer.

1999 Iberoamerican Mathematical Olympiad shortlist.
2000 Colombian Mathematical Olympiad, Problem 3.

Let n be an even positive integer. Find all triples of real numbers (x, y, z) such that

$$x^n y + y^n z + z^n x = x y^n + y z^n + z x^n.$$

1999 Colombian Mathematical Olympiad, Problem 5.

We want to color each cell of an 8×8 square board with one of four given colors, in such a way that the following condition holds: Whenever we place eight non-attacking rooks on the board in any possible way, and consider the cells that they are standing in, there are two cells of each color. How many different ways of doing this are there?

1995 Colombian Mathematical Olympiad, Problem 5.

Find all real numbers $\alpha \geq 0$ such that the equality

$$\lfloor (n-1)\alpha \rfloor \lfloor (n+1)\alpha \rfloor = \lfloor n\alpha \rfloor^2$$

holds for every integer $n \geq 2$.